# Lesson 9. Machine Scheduling

Example 1. The Markov Micromanufacturing Company has 9 production jobs it needs to process in the next 24 hours. The company has 4 identical machines that run in parallel. Each of these 9 jobs must be run on one of these machines **nonpreemptively**: that is, once a job is started on a machine, it must stay on that machine until it is completed. The processing times of these jobs are given below:

job	1	2	3	4	5	6	7	8	9
processing time (hours)	7	7	6	6	5	5	4	4	4

The company wants to minimize the **makespan**, or the completion time of the last job to finish processing.

- Let m be the number of machines — in this case, $m = 4$

- Suppose we schedule the jobs using the **longest processing time first (LPT)** rule:

    - First, schedule the m longest jobs on the m machines
    - Whenever a machine becomes free, put the longest unprocessed job on that machine

- Idea: LPT puts shorter jobs towards the end of the schedule, where they can be used to balance the loads on each machine

- For our problem, this yields a schedule that looks like this:
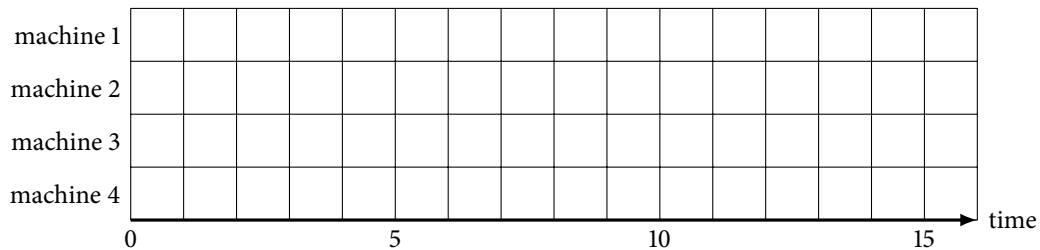
    - This kind of diagram is known as a **Gantt chart**

- Therefore, the makespan for the LPT schedule is

- It turns out that the makespan of an LPT schedule is <u>always</u> at most $33.\overline{3}\%$ larger than the minimum makespan

- So... can we do better?

- Let's formulate this problem as a dynamic program

- Stages:

- States in stage t (nodes):

- Decisions, transitions, and rewards/costs at stage t (edges):

- Shortest/longest path?

- Minimum makespan $\longleftrightarrow$

- Assignments of jobs to machines $\longleftrightarrow$

A   Problems

See the accompanying Jupyter Notebook for this lesson.